

Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

May, 2005

1. Linear Algebra

1.1 Given two square matrices $n \times n$ A and B conjugated, demonstrate that the outlines are equal.

1.2 Find a square matrix 4×4 A whose fourth power is null $A^4 = 0$, but in such a way that its minor powers A , A^2 , A^3 are null.

1.3 Calculate the sixth power of the following matrix:

$$A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$$

2. Calculus

2.1 Demonstrate the derivative of the function $\int_0^x \sin((x+t)^s) dt$ with respect to x .

2.2 Demonstrate that you have

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$

for each natural n .

2.3 Calculate the general solution of the following differential equation

$$yF(xy)dx = xG(xy)dy$$

3. Optional Problems

3.1 Demonstrate that the following series converge

$$\sum_{k=1}^{\infty} \frac{1}{k^3}$$

3.2 Demonstrate that the sphere is not homeomorphic to the circle.

3.3 Provide an example of an infinite abelian group with no isomorphic subgroup to integers.

3.4 Find the number of roots of $z^4 + 5z + 1 = 0$ inside the unit disc.