Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

June 2, 2006

Instructions: Solve all the problems in sections 1 and 2 and as many as you can solve in section 3. All solutions must be properly justified. The exam will last for 2 hours.

1. Linear Algebra

1.1 Let $A, B \in M_n(\mathbb{R})$ be two matrices n x n. Demonstrate that $\det(A + tB)$ a polynomial at t of grade $\leq n$ and calculate the quotient of t^n.

1.2 Calculate the determinant of the matrix n x n.

$$\begin{pmatrix} & & & 1 \\ & 0 & 1 & \\ & & \cdot & \\ & \cdot & & \\ & \cdot & 0 & \\ 1 & & & \end{pmatrix}$$

1.3 Find the orthonormal basis for the euclidian vector space V of th real polynomials of grade < = 2 with the scalar product:

$$\langle f, g \rangle = \int_0^1 f(x) \cdot g(x)\, dx\ .$$

2. Calculus

2.1 Calculate the vale of the expression $y_0 = f''(2) + f'(1) + f(0)$ if $f : \mathbb{R} \to \mathbb{R}$ is the function $f(x) = \int_0^x t^2 e^{t^2}\, dt.$

2.2 Determine if the series $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ is convergent or not.

2.3 Find the critical points of the function $f(x, y) = (\operatorname{sen} x) + y^2 - 2y + 1$ and determine the nature of them.

3. Optional Problems

3.1 Let H(n) be the real vector space of each hermitian complex matrices n x n of outline zero.

a) Which is the dimension of H(n) on R?

b) Prove that the Pauli's matrices $E_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $E_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ and $E_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ form a basis for H(2).

3.2 Two players A and B take turns to flip a coin. The winner of the game will be the one that gets tails. Assuming that A threw first, calculate the probability for B to win.

3.3 Evaluate the following integral using the residual method:

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\, dx$$

3.4 Let ℓ be a straight line on $\mathbb{R}^3$. Prove that $\mathbb{R}^3 \setminus \ell$ is not homeomorphic to $\mathbb{R}^3$.