Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

June 26, 2009

1. Linear Algebra

Solve all the problems in the two first sections.

1.1 Let it be

$$H_t := \begin{pmatrix} \cos 2\pi t & \cos \frac{\pi}{6}t \\ \operatorname{sen} 2\pi t & \operatorname{sen} \frac{\pi}{6}t \end{pmatrix} \text{ for } t \in \mathbb{R}.$$

Calculate the range of the matrix H_t for $0 \leq t < 12.$. Particularly, determine the t values for the matrix with range 1.

1.2 Calculate the determinant of the following matrix n x n.

$$\begin{pmatrix} 2 & 1 & 1 & \dots & 1 \\ 1 & 2 & 1 & \dots & 1 \\ 1 & 1 & 2 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 2 \end{pmatrix}$$

1.3 Let $A : V \to V$ be a linear transformation of a vector space of dimension 2, with one appropriate value λ and be E_λ the corresponding subspace of appropriate vectors. Prove that $Aw - \lambda w \in E_\lambda$, for each $w \in V$.

2. Calculus

2.1 Determine if the series $\sum_{n=1}^{\infty} nx^n$ converges for $0 < x < 1.$ If true, calculate the value of the such series.

2.2 Demonstrate that the function defined by

$$f(x) = x|x|, \quad x \in \mathbb{R}$$

is differentiable for each $x \in \mathbb{R}$ and that $f''(x)$ exists for each $x \neq 0$, but that $f''(0)$ does not exist. Draw graphs for f, f', f''.

2.3 What is the solution of $y'(t) = \frac{\operatorname{sen} t}{t}$, that satisfies $y(3) = -18$?

3. Additional Problems

3.1 Calculate the following integral:

$$\int_{-\infty}^{\infty} \frac{1}{x^4+1}.$$

3.2 Consider the space $C[0,1]$ with the norm $\|f\|_\infty = \sup_{x\in[0,1]} f(x)$.
Demonstrate that the norm $\|\|_\infty$ does not come from an internal product.

3.3 How many abelian groups of order 24 exist (except isomorphisms)

3.4 Let $SU(2)$ the group of unitary matrices 2 x 2 on C, with determinant 1:

$$SU(2) = \{A \in M_2(\mathbb{C}) \mid AA^* = I, \quad \det A = 1\}$$

with the topology of subspace of $M_2(\mathbb{C}) \simeq \mathbb{C}^4 \simeq \mathbb{R}^8$ and $A^* = \overline{A^t}$. Prove that SU(2) is homeomorphic to S^3, the unitary sphere in R^4.

3.5 Which of the following topological spaces are homeomorphisms among them? Justify your answer.

a) R
b) (0, 1)
c) [0, 1]
d) R^2