Centre for Research and Advanced Study at IPN

Department of Mathematics

Master's Degree Program Admission Examination

July, 2004

1. Linear Algebra

1.1 Find a square matrix 4 x 4 whose fourth potency A^4 be the zero matrix, but such that none of its other potencies $A,\ A^2$ and A^3 be the zero matrix

1.2 Consider the compound set for all polynomials with real coefficients nad grade greater or equals to two. Determine if this set is a vector space.

1.3 Givren the following vectors $v_1 = (3, 5, 11)$ and $v_2 = (2, 4, 8)$ in the Euclidian space $\mathbf{R}^3$. Calculate a unitary vector $v_3 \in \mathbf{R}^3$ that is orthogonal to v1 and to v2 s well.

2. Calculus

2.1 Consider the succession o real numbers $\{x_n\}_{n=0}^{\infty}$ recursively defined by the equation $x_{n+1} = x_n^2 + x_n - 3,$ for $n \geq 0.$ Determine the initial values x_0 that you need to get a constant succession $\{x_n\}_{n=0}^{\infty}$.

2.2 Calculate the derivative $\frac{df(t)}{dt}$ of the following function

$$f(t) = \int_0^t \frac{e^{st}}{s} ds$$

2.3 Calculate the general solution for the following differential equation

$$yF(xy)dx = xG(xy)dy$$

3. Optional problems

3.1 Demonstrate that the following series converges

$$\sum_{k=0}^{\infty} \frac{1}{k^2}$$

3.2 Determine the conformal transformation that sends the superior semi planar $\Im(z) > 0$ in the infinite strip $0 < \Im(z) < \pi.$

3.3 Calculate what the fundamental group is for the projective planar $\mathrm{RP}^2.$

3.4 Calculate the numerical value of the following integral:

$$2\pi i \int_{-\infty}^{\infty} \frac{dz}{z^4 + 1}$$