Centre for Research and Advanced Study at IPN

Department of Mathematics

Master's Degree Program Admission Examination

July 16, 2001

1. Linear algebra

1.1 Do real numbers r_1, r_2, r_3 and r_4 exist, such that polynomials

$p_1(x) = (x - r_1)(x - r_2),$

$p_2(x) = (x - r_2)(x - r_3),$

$p_3(x) = (x - r_3)(x - r_4),$

$p_4(x) = (x - r_4)(x - r_1),$

are linearly independent?

1.2 Determine the values of θ for the matrix

$$A = \begin{pmatrix} \cos\theta & -\mathrm{sen}\theta \\ \mathrm{sen}\theta & \cos\theta \end{pmatrix}$$

is defined by

1.3 Consider $\mathbb{R}^3$ as the internal product

$< \vec{u}, \vec{u} >= u_1 v_1 + 2u_2 v_2 + 3u_3 v_3$

a) Use the process of Gram-Schmidt to transform the basis:

$$\beta = \{(1,1,1), (1,1,0), (1,0,0)\}$$

in a orthonormal basis γ .

b) Obtain the change of basis matrix from β to γ.

2. Calculus

2.1 Calculate the solutions for the equation $F'(x) = 0,$ where $F : [0, \pi] \longrightarrow \mathbb{R}$ is the given function for

$$F(x) = x + \int_0^{\cos x} \sqrt{1 - x^2 dx.}$$

2.2 You have a circle and a square of areas A_1 and A_2, respectively. Determine the possible maximum of $A_1 + A_2$, subject to the condition of the sum of the perimeters is constant and equals to 10.

2.3

a) Demonstrate that the series $\sum_{n=1}^{\infty} (e/n)^n$ is convergent.
b) Use (a) to prove that the integral $\int_1^{\infty} (e^y / y^y) dy$ exists.
c) Using (b) and an appropriate substitution, tell if the following series is convergent or not.

$$\sum_{n=2}^{\infty} \frac{1}{(\log n)^{\log n}}$$

3. Optional problems

3.1 Let X be a vector space that is made of all the continuous functions $f : [0, 1] \longrightarrow \mathbb{R}$ with the norm of supreme $: \|f\| = \sup |f(x)|_{x \in [0,1]}$. Tell if x is compact.

3.2 List all abelian groups of order 24 (except isomorphism)

3.3 Let $D = \{z \in \mathbb{C} | |z| \leq 1\}$ be the closed unitary disc in the extended complex planar $\bar{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$. Find the constants a, b, c, d such that the function $f(z) = \frac{az+b}{cz+d}$ the interior of D to its exterior $\bar{\mathbb{C}} \backslash D$.