Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January 7, 2001

1. Linear Algebra

1.1 Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ defined by $T(x, y, z) = (x - y, y - z)$. Find the nucleus, nullity and range of T.

1.2 Let P_2 be the vector space of real polynomials of less grade or equals to

$2 : P_2 = \{a + bx + cx^2 | a, b, c \in \mathbb{R}\}$. The usual or canonical basis of P_2 is given by the polynomials 1, x, x^2.

a) Find the basis of P_2 different that the usual and express the polynomial $a + bx + cx^2$ as a linear combination of this basis.
b) Calculate the change of basis matrix.
c) What is the dimension of dual space of P_2?

1.3 Let $t_1, t_2, t_3 \in \mathbb{R}$ be three different real numbers and P_2 as the previous problem. For I = 1, 2 , 3 be

$$T_i : P_2 \longrightarrow \mathbb{R}$$

The function given by $T_i(p) = p(t_i)$ that is, the evaluation of the polynomial p on ti.

a) Prove that the functions T_1 T_2 and T_3 are linear transformations.
b) Prove that the functions T_1T_2 and T_3 are linearly independent in the dual space of P_2 and therefore a basis for the dual space.

2. Calculus

2.1 Let $a, b \in \mathbb{R}$ such $a > 2b > 0$ and be $F : [0, \frac{\pi}{3}] \longrightarrow \mathbb{R}$ the function given by:

$$F(x) = \int_0^{\pi x} \frac{d\theta}{a\cos\theta - b\,\mathrm{sen}\theta}$$

Find a critical point of the function $F(x) - \frac{\sqrt{2\pi}}{(a-b)}x$ in the open interval $(0, \frac{(\pi)}{3})$

2.2 Tell if the following series are convergent:

$$(a) \sum_{n=1}^{\infty} \frac{2^{n-1}}{n^n} \qquad (b) \sum_{n=1}^{\infty} \frac{1}{n^2} \operatorname{sen}(\pi/n)$$

Prove that it is impossible to place $x = f(x)g(x)$ where f and g are derivable and f(0) = g(0) = 0

3. Optional Problems

3.1 Let $f : X \longrightarrow Y$ a continuous bijection between two topological spaces. Prove that if X is compact and Y is Hausdorff then f is homeomorphism.

3.2 Prove that if a in group G ever item is it own reserve, then G is abelian.

3.3 Suppose that $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$ has partial derivatives or order two. Tell which of the following identities are true:

$$\text{(a)} \;\; \nabla \times (\nabla f) = \vec{0}$$

$$\text{(b)} \;\; \nabla .(\nabla \times f) = 0$$

$$\text{(c)} \;\; \nabla .(\nabla f) = 0$$

$$\text{(d)} \;\; \nabla \times (\nabla . f) = \vec{0}$$

3.4 Calculate the following integral $\int_{-\infty}^{\infty} \frac{dx}{x^6+1}$