Centre for Research and Advanced Study at IPN

Department of Mathematics

Master's Degree Program Admission Examination

January 8, 2006

1. Linear Algebra

1.1 Evaluate the determinant of

$$A = \begin{pmatrix} 2 & 0 & 0 & 3 & 1 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 \\ 2 & 2 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{pmatrix}$$

1.2 Let $B = \{v_1, \ldots, v_n\} \subset \mathbb{R}^n$ be such that $\langle v_i, v_j \rangle = 0$ each that $i \neq j$. If for each i we know that $v_i \neq 0$ demonstrate that B is a basis of $\mathbb{R}^n$.

1.3 Let $V = \{f(x) \in \mathbf{R}[x] | deg(f) \leq 5\}$. Let $T : V \longrightarrow \mathbf{R}^6$ given by $T(f) = (f(0), f(1), f(2), f(3), f(4), f(5))$

Demonstrate that T is linear and find its nucleus.

2. Calculus

2.1 Let $f : \mathbf{R} \longrightarrow \mathbf{R}$ be given by

$$f(x) = \int_{\sin x}^{x^3} t(t-3)dt$$

Calculate $f'(x)$

2.2 Let $f : [0,1] \longrightarrow [0,1]$ be differentiable. Demonstrate that f is uniformly continuous.

2.3 Demonstrate that $f(x) = \frac{1}{3}x^3 + \frac{5}{3}x + 2$ is reversible.

3. Optional problems

3.1 Make a decision what is more likely when throwing a dice repeatedly. Obtain a 6 in six throws or at least two 6's in twelve throws.

3.2 Let G be a finite group and H a subgroup of index two. Prove that H is normal on G.

3.3 Demonstrate that if an integer, holomorphic function of complex planar satisfies:

$$|f(z)| \leq C|z|^n$$

for a positive constant C and a natural n, then f is a polynomial and its grade is less than or equals to n.

3.4 Demonstrate that the closed interval $X = [0, 1]$ is compact, that is, demonstrate that open cover of X admits a finite subcover.