Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

January 13, 2003

1. Linear Algebra

1.1 Let $\mathrm{M}_n(\mathbb{C})$ the vector space of matrices n x n on complex numbers.

1.1.a Prove that the function $F(A,B) = tr(AB^*)$ defines an internal product in $\mathrm{M}_n(\mathbb{C})$. Here B* denotes the conjugated transverse of matrix B.
1.1.b Find the orthogonal basis of $\mathrm{M}_n(\mathbb{C})$ in respect to this internal product.

1.2 Prove that for every $B \in \mathrm{M}_n(\mathbb{C})$, the linear operator $T_B : \mathrm{M}_n(\mathbb{C}) \longrightarrow \mathrm{M}_n(\mathbb{C})$ possess a determining null where T_B is defined by $T_B(A) = AB - BA$.

1.3 Le V be a real vector space of finite dimension and T an linear operator on V. Let c1, c2, ..., ck be the appropriate values different to T and for I = 1, 2, ..., k, be Wi an appropriate space related to ci. Prove that the equivalency of the following statements:

a. The matrix associated to T in respect to any basis of V is diagonal.
b. The characteristic polynomial of shape

$(x-c_1)^{d_1}(x-c_2)^{d_2}\cdots(x- \ c_k)^{d_k}$ where dim (Wi) = di for I = 1,2, ..., k.

2. Calculus

2.1 Prove that the D'Alembert criteria for series convergence: "Every series $\sum_{i\geq 1} a_n$ of positive terms that satisfies the condition:

$$\lim_{n\longrightarrow\infty} \frac{a_{n+1}}{a_n} < 1$$

is convergent.

2.2 Let *f* be a differentiable function in the closed interval [a, b]

2.2.a Is f' necessarily continuous? Argue (by giving a negative answer, proving a positive answer)

2.2.b Suppose the existence of a point C with f'(a) < C f'(b). Analyze the function g defined by g(x) = f(x) – C (x – a) to deduce the existence of a point xo with $a < x_0 < b$ and such $f'(x_0) = C$.

2.3 Find a derivative of the function F defined in [0, 1] by the formula:

$$F(x) = \int_{2-x}^{2+x} \ln(\sqrt{t})dt.$$

3. Optional problems

3.1 Let $\phi : \mathbb{R}^2 \longrightarrow \mathbb{R}$ the projection $\phi(x, y) = x$. Is ϕ a closed function? Or an open one?

3.2 Let G be a finite group and subgroup H (not necessarily normal) of G. For $a \in G$ be $f(a)$ the minimum positive m such a^m such $a^m \in H$. Prove that $f(a)$ is a divisor of order a in G.

3.3 For n > = 0 recursively evaluate the undefined integral $\int e^{-x}x^n dx.$

3.4 Prove that every open set of a real straight line is the union to the enumerable sum of open intervals.