Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 20, 1999

1. Linear Algebra

1.1 Let A be a matrix of order n and denote by A' its transpose. Prove that if A' = -A and n is an odd number then *det* A =0.

1.2 Let W be the subspace of $\mathbb{R}^3$ generated by

$$\begin{aligned}\mathcal{B} &= \{\alpha_1 = (2,1,1), \alpha_2 = (-1,2,0)\\ &\quad \alpha_3, = (7,-4,2), \alpha_4 = (1,1,1)\}.\end{aligned}$$

Determine a basis for W contained in $\mathcal{B}$.

1.3 Consider the following matrix:

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Determine the appropriate values for A and a basis for the subspaces of corresponding appropriate vectors.

2. Calculus

2.1 Graph the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = x^3 - 3x$ noting local extremes, inflexion points and intervals within concavity or convexity.

2.2 Let $g : (1, \infty) \longrightarrow \mathbb{R}$ be given by

$$g(x) = \int_1^{x^3} \cos\,(1 + \sqrt{t})dt$$

Calculate: $g'(x)$.

2.3 Determine if the following series is convergent and justify your answer:

$$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$

3. Optional problems

3.1 Let $\{Vi\}_{i=l}^{\infty}$ be an arbitrary succession of open sets in $\mathbb{R}^n$. Is $\bigcap_{1=l}^{\infty} Vi$ always an open set in $\mathbb{R}^n$. Justify your answer.

3.2 Let $f_n : (0, \infty) \longrightarrow \mathbb{R}$ be given by
$f_n(x) = x^n, \text{con}\, n = \ldots, -2, -1, 0, 1, 2, \ldots.$

What values for n make f_n uniformly continuous? Justify your answer.

3.3 Let (X, d) a metric space and define for $x, y \in X$

$$\hat{d}(x, y) := \text{mín}\{1, d(x, y)\}$$

Prove that $\hat{d}$es is a metric on X that determines the same open sets.

3.4 Let $(\mathbb{Z}_n, +)$ be the additive group for the whole integer module n. Is the Cartesian product $\mathbb{Z}_2 \times \mathbb{Z}_4$ isomorphic to $\mathbb{Z}_8$?. Justify your answer.