Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 7, 2008

1. Linear Algebra

1.1 Consider the following vectors:

v1 = (1,1,1,a), v2 = (1,2,3,a), v2= (b,-1,0,1), v4=(0,b,0,0)

where a and b are real numbers. Determine the maximum dimension and minimum generated space by {v1, v2, v3, v4}.

1.2 Provide an example of a 3 x 3 matrix with real entries that is not similar to a diagonal matrix.

1.3 Find the basis for the null space of the matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix}.$$

2. Calculate

2.1 Calculate the following limit

$$\lim_{x\to\infty} \operatorname{sen}(\frac{a}{x}).$$

2.2 Let y1, y2, y3 be particular solutions for the linear differential equation of first order: $y'(x) = a(x)y(x).$

Prove that the following expression is constant:

$$\frac{y_3(x) - y_2(x)}{y_3(x) - y_1(x)}$$

2.3 Find local maxima and minima for the following function:

$$y(x) = \int_0^x \frac{\operatorname{sen} t}{t}\, dt.$$

3. Optional problems

3.1 Prove that the integer group of order 4 is isomorphic to Z^4 or to Z^2 x Z^2
3.2 Provide an example of a succession of functions on L2(R) that converges to 0 with a norm on L2(R)

3.3 Let A be a connected set, open and closed in a metric space X. Prove that A is a connected component of X.

3.4 Prove that each holomorphic bijection between two discs of the complex planar if formed by

$$f(z) = \frac{az+b}{cz+d},$$

for some constants a,b,c,d
Suggestion: Use Schwarz lemma.