Centre for Research and Advanced Study at IPN

Department of Mathematics

Master' Degree Program Admission Examination

August 13, 2001

1. Linear Algebra

1.1 Consider the following matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

a) Calculate the range for A
b) Let $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$ be a linear transformation of which matrix, in respect of, a canonical basis R4 is given by A. Calculate a basis for nucleus of T.

1.2 Let $T : V \longrightarrow V$ be a linear transformation.

a) Assuming that T is a reversible, prove that λ is an appropriate value of T if and only if λ^{-1} is proper value of T^{-1}.
b) If V is in a finite dimension, prove that T is reversible if and only if $\vec{0}$ is not a proper value of T.

1.3 Determine if the following matrix is diagonalizable:

$$A = \begin{pmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{pmatrix}$$

If true, find a matrix Q in such a way that Q-1 AQ be a diagonal matrix.

2. Calculus

2.1 Find the derivative of function:

$$F(x) = \int_a^b \frac{x^2}{1 + 2\text{sen}^3 t + \text{sen}^6 t} dt$$

2.2 Consider the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by:

$$A = \begin{cases} \text{sen}(\frac{1}{x}) & \text{si } x \neq 0 \\ 0 & \text{si } x = 0 \end{cases}$$

a) Decide if f is continuous in x_o when $x_o \neq 0$

b) Let $\{y_n\}_{n=1}^{\infty}$ be a succession given by $y_n = f(\frac{1}{n\pi})$. Calculate

$$\lim_{n \longrightarrow \infty} y_n.$$

c) Decide if f is continuous in x = 0.

2.3 Let F and g continuous functions of R in R. It is true that f(x) = g(x) for all $x \in \mathbb{R}$ if and only if si $f(y) = g(y)$ for all $y \in \mathbb{Q}$?.

3. Optional problems

3.1 Say if function $f : \mathbb{C} \longrightarrow \mathbb{C}$ given by $f(z) = \bar{z}$ is analytical in C.

3.2 Consider the norm in $\mathbb{R}^n$ given by $\|\vec{x}\|_1 = |x_1| + \ldots + |x_n|$. Compare the topologies in $\mathbb{R}^n$ induced by $\|\cdot\|_1$ and the Euclidean norm.

3.3 A truncated icosahedrons (i.e. a soccer ball) is a polyhedron whose faces are regular pentagonal and hexagonal. How many pentagonal faces does it have? Suggestions: Remember that the Euler characteristic of the sphere is 2.

3.4 Let $G_{m,n} = Hom(\mathbb{Z}/m, \mathbb{Z}/n)$ be the group of all homomorphisms in h: $\mathbb{Z}/m \longrightarrow \mathbb{Z}/n$ with the operation of sum of functions

$$(f + g)(x) \quad := \quad f(x) + g(x).$$

Calculate the order of $G_{m,n}$.