Sam’s 80

A Conference to Celebrate Sam Gitler’s 80th Birthday

Schedule:

September 25 (Wednesday)

10:00 – 10:50	**Opening ceremony**
10:50 – 11:40	**Don Davis** (Lehigh University) Stable geometric dimension: Old work with Mark and Sam (and Martin)
11:40 – 12:00	**COFFEE BREAK**
12:00 – 12:50	**Fred Cohen** (University of Rochester) An excursion into moment-angle complexes, polyhedral products, and their applications
12:50 – 13:40	**Mikiya Masuda** (Osaka City University) Toric origami manifolds in toric topology
13:40 – 15:30	**LUNCH**
15:30 – 16:20	**Bill Browder** (Princeton University) How big a finite group can act freely on a product of spheres?
16:20 – 17:10	**Ralph L. Cohen** (Stanford University) Gauge theory, loop groups, and string topology
17:10 – 17:30	**COFFEE BREAK**
17:30 – 18:20	**Soren Galatius** (Stanford University) Manifolds and moduli spaces

September 26 (Thursday)

10:00 – 10:50	**Jesús González** (Cinvestav-IPN) Topological robotics
10:50 – 11:40	**Martin Bendersky** (Hunter College, CUNY) Structure of The Polyhedral Product and Related Spaces
11:40 – 12:00	**COFFEE BREAK**
12:00 – 12:50	**Dennis Sullivan** (SUNY, Stony Brook) From Topology to Analysis
12:50 – 13:40	**Douglas C. Ravenel** (University of Rochester) A Solution to the Arf-Kervaire Invariant Problem
13:40 – 15:30	**LUNCH**
15:30 – 16:20	**Kee Yuen Lam** (University of British Columbia, Canada) Solution of the Yuzvinsky conjecture for certain types of matrices
16:20 – 17:10	**Victor M. Buchstaber** (Steklov Mathematical Institute, Russia) Toric topology and Grassmann manifolds
17:10 – 17:30	**COFFEE BREAK**
17:30 – 18:20	**Soren Galatius** (Stanford University) The method of scanning

September 27 (Friday)

10:00 – 10:50	**Ernesto Lupercio** (Cinvestav-IPN) Non-commutative Toric Varieties
10:50 – 11:40	**Tony Bahri** (Rider University) A generalization of the standard topological construction of toric manifolds and applications involving various operations on simplicial complexes
11:40 – 12:00	**COFFEE BREAK**
12:00 – 12:50	**Taras Panov** (Moscow State University) Complex geometry of moment-angle manifolds
12:50 – 13:40	**Alberto Verjovsky** (UNAM) Poincaré theory for compact abelian one-dimensional solenoidal groups
13:40 – 15:30	**LUNCH**
15:30 – 16:20	**Santiago López de Medrano** (UNAM) Projective moment-angle complexes (first steps)
16:20 – 17:10	**Alejandro Adem** (University of British Columbia) A classifying space for commutativity in Lie groups
17:10 – 17:30	**COFFEE BREAK**
17:30 – 18:20	**Soren Galatius** (Stanford University) Homological stability for moduli spaces

Titles and Abstracts

A classifying space for commutativity in Lie groups

Alejandro Adem
University of British Columbia
adem@pims.math.ca

In this talk we consider a space $B_{com}G$ assembled from commuting elements in a Lie group G. We describe homotopy-theoretic properties of these spaces using homotopy colimits, and their role as a classifying space for transitionally commutative bundles. We prove that they are loop spaces and define a notion of commutative K-theory for bundles over a finite complex X which is isomorphic to $[X, Z \times B_{com}G]$. We compute the rational cohomology of $B_{com}G$ for G equal to any of the classical groups $U(n)$, $SU(n)$ and $Sp(n)$, and exhibit the rational cohomologies of $B_{com}U$, $B_{com}SU$ and $B_{com}Sp$ as explicit polynomial rings. This is joint work with Jose Manuel Gomez.

A generalization of the standard topological construction of toric manifolds and applications involving various operations on simplicial complexes

Tony Bahri
Rider University
bahri@rider.edu

The Davis-Januszkiewicz construction on simple polytopes and tori is reinterpreted in a way which allows for generalization. Application is made to a construction of infinite families of toric manifolds which requires simpler combinatorial input than usual. Also discussed will be certain generalizations involving the iterated operations on simplicial complexes introduced recently by A. Ayzenberg. A report of joint work with Martin Bendersky, Fred Cohen and Sam Gitler.

Structure of The Polyhedral Product and Related Spaces

Martin Bendersky
Hunter College, CUNY
mbendersky1@gmail.com

I will talk about recent work with Tony Bahri, Fred Cohen and Sam Gitler. I will give a survey talk on the topology of polyhedral products. The PPF generalizes the moment angle complex (from toric geometry) the Stanley Reisner ring (from combinatorics) and constructions of Davis and Januszkiewicz.

How big a finite group can act freely on a product of spheres?

Bill Browder
Princeton University
browder@math.princeton.edu

Since P A Smith's theory appeared almost 75 years ago (perhaps the first place in the algebraic topology literature where the hypothesis "p is prime" is essential), it has remained a challenge to extend his results to spaces more complicated than spheres.

The Borel Seminar of 1957 revealed the strong relation of Smith's method to cohomology of groups, and A Heller in 1958 solved the problem (of my title) for the product of two spheres, while G Carlsson in 1980 solved it for spheres of the same dimension. More recently B Hanke produced a solution for the case where the prime p is very large compared to the dimension of the space.

In this talk I will describe some different points of view which lead to new results such as:

Theorem A: Let $G = (Z_2)^r$ act "freely" on X whose homotopy type is the same as that of $(S^3)^a \times (S^n)^b$, n odd and trivially on homology. Then r is less than or equal to $a + b$.

Theorem B: Let $G = (Z_p)^r$ act "freely" on X of the homotopy type of $(S^1)^a \times Y$ and trivially on homolgy. Then a subgroup K of G of rank greater than or equal to $r - a$, acts "freely" on Y.

The definition of "free" action is purely cohomological and generalizes the usual properties from finite dimensional to a larger class of spaces which may be infinite dimensional.

Corollary 1: If G acts "freel" on X homotopy equivalent to $(S^1)^a \times (S^n)^b$ then r is less than or equal to $a + b$.

Corollary 2: If G acts as in B and Y has the homology type of $S^k \times S^m$ then r is less than or equal to $a + 2$.

The proof of Theorem A demonstrates how the Steenrod algebra must be strongly involved in the solution of this problem.

Toric topology and Grassmann manifolds

Victor M. Buchstaber
Steklov Mathematical Institute, RAS, Moscow, Russia
buchstab@mi.ras.ru

We introduce the class of toric $(2n, k)$-manifolds, which are special class of smooth manifolds M^{2n} equipped with a smooth action of the compact torus T^k and a moment map $\mu : M^{2n} \to P^k$, where $P^k \subset \mathbb{R}^k$ is a convex polytope. Any $(2n, 1)$-manifold is homeomorphic to the standard sphere S^{2n}. The class of $(2n, n)$-manifolds coincides with the class of quasitoric manifolds which is one of the key objects of toric topology. In particular any algebraic smooth toric variety is a $(2n, n)$-manifold. The symplectic $2n$-dimensional manifolds with a Hamiltonian action of T^k give to us an important class of $(2n, k)$-manifolds. Special attention we devote to the description of $(2n, k)$-structure on the complex Grassmann manifolds $G_{k+1,l}$, where $n = l(k+1-l)$. We discuss the well known problem about the structure of the orbit space $G_{k+1,l}/T^k$ and the properties of the classical moment map $\mu : G_{k+1,l} \to \Delta_{k+1,l}$, where $\Delta_{k+1,l}$ is the hypersimplex. Our results are amed for new applications and development of toric topology methods. The talk is based on recent results obtained jointly with Svjetlana Terzić.

An excursion into moment-angle complexes, polyhedral products, and their applications

Fred Cohen
University of Rochester
cohf@math.rochester.edu

Definitions, and applications of polyhedral products will be given. This talk represents joint work with A. Bahri, M. Bendersky, and S. Gitler.

Gauge theory, loop groups, and string topology

Ralph L. Cohen
Stanford University
rlc@stanford.edu

In this talk I will describe a joint project with John Jones, in which we relate the gauge theory of a principal bundle $G \to P \to M$ to the string topology spectrum of the principal bundle, $\mathcal{S}(P)$. This spectrum has, as its homology the homology of the adjoint bundle, $P^{Ad} = P \times_G G$, where G acts on itself by conjugation. In this study G can be any topological group. In the universal case when P is contractible, $P^{Ad} \simeq LM$, the free loop space, and this spectrum realizes the original Chas-Sullivan homological structure. One of our main results is to identify the group of units of $\mathcal{S}(P)$, and to relate it to the gauge group of the original bundle, $\mathcal{G}(P)$. We import some of the basic ideas of gauge theory, such as the action of the gauge group on the space of connections, to the setting of principal fiberwise spectra over a manifold, and show how it allows us to do explicit calculations. We also show how, in the universal case, an action of a Lie group on a manifold yields a representation of the loop group on the string topology spectrum. We end by discussing a functorial perspective, which describes a sense in which the string topology spectrum $\mathcal{S}(P)$ of a principal bundle is the "linearization" of the gauge group $\mathcal{G}(P)$.

Stable geometric dimension: Old work with Mark and Sam (and Martin)

Don Davis
Lehigh University
dmd1@lehigh.edu

I am going to talk about joint work with Mark and Sam in the late 1970's which was redone from a different perspective in the 21st century in joint work with Mark and Martin. It yields a nice formula for the geometric dimension of a vector bundle of order 2^e over RP^n when n is large.

Series: The topology of moduli spaces

Soren Galatius
Stanford University
galatius@stanford.edu

Lecture 1: Manifolds and moduli spaces

Manifolds arise in many areas of mathematics and science, for example as the solution set of certain equations, the state space of a physical system, or as a model for the universe. The study of manifolds and their properties has been a driving force behind *topology* for many decades. In this lecture I will explain the idea that the collection of all manifolds may itself be regarded as a kind of space, a *moduli space*, and that such spaces have interesting and useful properties.

Lecture 2: The method of scanning

Scanning is a remarkably successful method for understanding the topology of moduli spaces. I will explain the method with some examples, some classical and some more recent.

Lecture 3: Homological stability for moduli spaces

Moduli spaces of manifolds often arise in families, indexed by some number g. For example, there is a moduli space of genus g surfaces for each natural number g, and similar patterns appear for manifolds of higher dimension. In recent joint work with Oscar Randal-Williams, we prove that such moduli spaces often exhibit *homological stability*: the cohomology of the moduli space is independent of g, in a range of degrees that grows linearly with g.

Topological robotics

Jesús González
Cinvestav-IPN
jechucho@gmail.com

I will survey recent developments in the application of techniques from algebraic topology to the path motion planning problem in robotics.

Solution of the Yuzvinsky conjecture for certain types of matrices

Kee Yuen Lam
University of British Columbia, Canada
lam@math.ubc.ca

The Yuzvinsky conjecture is concerned with a problem in the combinatorics of matrix coloring. Given a blank matrix M of r rows and s columns, one seeks to color its entries using n colors such that

1. each row should have no repeated colors,

2. each column should have no repeated colors, and
3. each 2 by 2 submatrix of M contains either two colors or four colors.

In 1981 S. Yuzvinsky conjectured that the minimal number n of colors required is given by a certain numerical function on r and s that is already familiar in dyadic arithmetic. This coloring problem has a topological origin in the study of geometric dimension of vector bundles. It also has an algebraic origin in the study of sums of squares. In this talk I shall explain these connections, and present a partial solution of Yuzvinsky's conjecture based, indeed, on ideas from algebra and topology.

Projective moment-angle complexes (first steps)

Santiago López de Medrano
UNAM
santiago@matem.unam.mx

Let K be a simplicial complex on m vertices and $\mathcal{Z}$ the associated moment-angle complex (see [B-B-C-G] for notation and a very general construction):

$$\mathcal{Z} = \mathcal{Z}(K) = \mathcal{Z}(K; (D^2, S^1))$$

The torus T^m acts on $\mathcal{Z}$. Take the diagonal S^1 in T^m and the restriction to it of the action. We will call the quotient a *projective moment-angle complex*:

$$\mathcal{PZ} := \mathcal{Z}/S^1.$$

(Analogous definitions can be given for the *real* versions of these objects: the quotients $\mathcal{Z}(K; (D^1, S^0))/\mathbf{Z}_2$, for the action of the diagonal $\mathbf{Z}_2 \subset \mathbf{Z}_2^m$).

When K is the dual of the boundary of a simple convex polytope, $\mathcal{Z}$ is a smooth manifold (named *moment-angle manifold*) and so is $\mathcal{PZ}$. Examples:

a) $\mathcal{Z}$ can be a product of odd-dimensional spheres with the usual torus action on each factor: $\mathcal{PZ}$ is then a *complex projective product space* (cf. [D]).

b) If $\mathcal{Z}$ is odd-dimensional, $\mathcal{PZ}$ is a so-called LV-M manifold (see the survey [M-V]) and has a complex structure with a natural family of deformations.

In this talk I will give some general topological properties of projective moment-angle complexes and describe the topology of some projective moment-angle manifolds. For the moment I have more questions than answers, but I am sure that suggestions from the many experts participating in the Sam80 conference will be of much help for developing this subject.

[B-B-C-G], A. Bahri, M. Bendersky, F. R. Cohen, and S. Gitler, *The polyhedral product functor: a method of computation for moment-angle complexes, arrangements and related spaces*, Adv. in Mathematics 225(2010), 1634-1668.

[D], D. Davis, *Projective product spaces*, J. of Topology 3(2010), 265-279.

[M-V], L. Meersseman and A. Verjovsky, *Sur les variétés LV-M*, Contemporary Mathematics 475(2008) 111-134.

Non-commutative Toric Varieties

Ernesto Lupercio
Cinvestav-IPN
`elupercio@gmail.com`

In this talk I will introduce non-commutative toric varieties using LVM-theory and variation on diffeologies as tools. Then I will expose some of the properties of such objects. Finally I will describe the basic structure of the moduli space of NC Toric varieties.

Toric origami manifolds in toric topology

Mikiya Masuda
Osaka City University
masuda@sci.osaka-cu.ac.jp

Cannas da Silva-Guillemin-Pires [1] introduced the notion of a toric origami manifold, which weakens the notion of a toric symplectic manifold, and they show that toric origami manifolds bijectively correspond to origami templates via moment maps, where an origami template is a collection of Delzant polytopes with some folding data. In this talk we discuss the topology of toric origami manifolds.

Bibliography

1. A. Cannas da Silva, V. Guillemin and A. R. Pires, *Symplectic Origami*, IMRN 2011 (2011), 4252–4293, arXiv:0909.4065.
2. T. Holm and A. R. Pires, *The topology of toric origami manifolds*, arXiv:1211.6435.
3. M. Masuda and S. Park, *Toric origami manifolds and multi-fans*, arXiv:1305.6347.

Complex geometry of moment-angle manifolds

Taras Panov
Moscow State University
tpanov@mech.math.msu.su

Moment-angle complexes are spaces acted on by a torus and parametrised by finite simplicial complexes. They are central objects in toric topology, and currently are gaining much interest in homotopy theory. Due the their combinatorial origins, moment-angle complexes also find applications in combinatorial geometry and commutative algebra.

After an introductory part describing the general properties of moment-angle manifolds and complexes we shall concentrate on the complex-analytic aspects of the theory.

Moment-angle manifolds provide a wide class of examples of non-Kähler compact complex manifolds. A complex moment-angle manifold Z is constructed via a certain combinatorial data, called a complete simplicial fan.

In the case of rational fans, the manifold Z is the total space of a holomorphic bundle over a toric variety with fibres compact complex tori. By studying the Borel spectral sequence of this holomorphic bundle, we calculate the Dolbeault cohomology and Hodge numbers of Z.

In general, a complex moment-angle manifold Z is equipped with a canonical holomorphic foliation F and an algebraic torus action transitive in the transverse direction. Examples of moment-angle manifolds include the Hopf manifolds, Calabi-Eckmann manifolds, and their deformations.

We construct transversely Kähler metrics on moment-angle manifolds, under some restriction on the combinatorial data. We prove that all Kähler submanifolds in such a moment-angle manifold lie in a compact complex torus contained in a fibre of the foliation F. For a generic moment-angle manifold in its combinatorial class, we prove that all its subvarieties are moment-angle manifolds of smaller dimension. This implies, in particular, that its algebraic dimension is zero.

This is joint work with Yuri Ustinovsky and Misha Verbitsky.

A Solution to the Arf-Kervaire Invariant Problem

Douglas C. Ravenel
University of Rochester
`dravenel@z.removethis.rochester.edu`

The Arf-Kervaire invariant problem arose from Kervaire-Milnor's classification of exotic spheres in the early 1960s. Browder's theorem of 1969 raised the stakes by connecting it with a deep question in stable homotopy theory. In 2009 Mike Hill, Mike Hopkins and I proved a theorem that solves all but one case of it. The talk will outline the history and background of the problem and give a brief idea of how we solved it.

From Topology to Analysis

Dennis Sullivan
SUNY, Stony Brook
dennis@math.sunysb.edu

Consider dividing space into cubes and subdividing again and again. The cells of each subdivision after the first have a natural partial semi group structure. This gives an associative algebra structure on the real cochains at each level for which the coboundary operator is a derivation. Natural transversal inequalities imply these various products have a limit under subdivision. The limit is graded commutative and is consistent with the wedge product of differential forms.

Each subdivision has a poincare dual cell operator and adding in dual tangential inequalities imply these converge to an operator consistent with the hodge star.

One may now write a system of finite dimensional ODEs associated to infinite dimensional PDEs. One may use this on two ways:
First, one may try to study the PDE for solvability or blow up using these nonlinear ODE approximations and the finite dimensional theory of dynamics.
Second, one may use these ODEs as effective models of the physical process the PDE is meant to model exactly. This is reasonable even when the PDE itself is divergent or intractable. The motivating example is the Navier Stokes model of incompressible fluid motion.

Poincaré theory for compact abelian one-dimensional solenoidal groups

Alberto Verjovsky
UNAM
alberto@matcuer.unam.mx

In 1881, H. Poincaré introduced an invariant $\rho(f)$ of topological conjugation for homeomorphisms $f : S^1 \to S^1$ of the unit circle S^1 called the rotation number of f.

He then proved a remarkable topological classification theorem for the dynamics of any orientation-preserving homeomorphism f has a periodic orbit if and only $\rho(f)$ is rational. If the rotation number $\rho(f)$ is irrational, then f is semiconjugate to an irrational rotation $R(f)$. The semiconjugacy is actually a conjugacy if the orbits of f are dense. This study has been one of the most fruitful subjects in the theory of dynamical systems as witnessed by the works of A.N. Kolmogorov, V.I. Arnold, J. Moser, M.R. Herman, A.D. Brjuno, J.C. Yoccoz, among others.

In this talk I describe how the rotation number can be generalized to the case of homeomorphisms of a general compact abelian one-dimensional solenoidal group, which is also a onedimensional foliated space; specifically, the theory is developed for the algebraic universal covering space of the circle. Poincaré's dynamical classification theorem is also generalized to homeomorphisms of solenoids whose rotation element is an irrational element (i.e., monothetic generator) of the given group.
