Heat kernel bounds for isotropic-like Laplacians on ultrametric spaces

Abstract

Let (X, d, m) be a proper ultrametric space equipped with a measure m. Given a symmetric measurable function $J(x, y)$ we consider the integral operator

$$L^J f(x) = \int (f(x) - f(y))J(x, y)dm(y)$$

defined on the set D of test functions, i.e. all locally constant functions f having compact support. We assume that m has full support and that the function $J(x, y)$ is uniformly in x, y comparable to a certain isotropic function $I(x, y)$. Under some reasonable assumptions on the function $I(x, y)$ the operator $(-L^J, D)$ is essentially self-adjoint, extends in $L^2(X, m)$ as a self-adjoint Markov generator and its Markov semigroup $exp(-tL^J)$ admits a continuous transition density (heat kernel) $p^J(t, x, y)$ w.r.t. m. Moreover, the function $p^J(t, x, y)$ is uniformly comparable in t, x, y to the transition density $p^I(t, x, y)$ associated with the isotropic Markov semigroup $exp(-tL^I)$ - an ultrametric version of the well-known Aronson's theorem for uniformly elliptic operators in eucledian spaces.