

 **Sixth International Conference on p-adic Mathematical
Physics and its Applications (Mexico.p-adics2017)**
CINVESTAV, Mexico City, October 23rd-27th, 2017



Abstracts

October 23rd-27th, 2017

Hour	Monday 23 rd	Tuesday 24 th	Wednesday 25 th	Thursday 26 th	Friday 27 th	
8:30 - 10:00	BREAKFAST					
10:00-11:00	Registration and Opening	Conference #6	Conference #12	Conference #18	TRIP TO MEXICO CITY	
11:00 - 11:30	COFEE BREAK					
11:30-12:30	Conference #1	Conference #7	Conference #13	Conference #19		
12:45-13:45	Conference #2	Conference #8	Conference #14	Conference #20		
14:00 - 15:00	LUNCH					
15:00 - 16:00						
16:00-17:00	Conference #3	Conference #9	Conference #15	Conference #21		
17:00 - 17:30	COFEE BREAK					
17:30-18:30	Conference #4	Conference #10	Conference #16	Conference #22		
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19:40-20:40	DINNER					

Conference #1. Debashis Goshal
An Ultrametric Route to Berry-Keating

Conference #2. Luis Gorostiza
Random Walks And Percolation In A Hierarchical Lattice

Conference #3. Miriam Bocado, Hugo García Compeán
On p -adic String Amplitudes in the limit p approaches to one

Conference #4. Juan Manuel Burgos
Ultrametric space in Teichmüller theory and nonperturbative String theory

Conference #5. Alexey Koshelev
 p -adic cosmology to become quantum gravity

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 p -Genetic Code and Ultrametric Bioinformation

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Conference #13. Vladimir Osipov
Ultrametric models in theory of symbolic sequences

Conference #14. Alain Escassut
New applications of the p -adic Nevanlinna Theory, in a field K, inside an open disk or out of a hole

Conference #15. Abdelmalek Abdesselam
Recent progress in p-adic quantum field theory

Conference #16. Khodr Shamseddine
One-variable and Multi-variable Integral Calculus over the Levi-Civita Field and Applications

Conference #17. Anatoly Kochubei
p-Adic Analogue of the Porous Medium Equation

Conference #18. Goran Djordjevic
Nonarchimedean and noncommutative aspects of the interior of the Schwarzschild black hole and signature change

Conference #19. Ingmar Akira Saberi
p-adic approaches to discretizing holography

Conference #20. José Aguayo
Representation Theory for operators on Free Banach space of countable type

Conference #21. Zoran Rakić
Path integrals on real, p-adic, and adelic spaces

Conference #22. Sergei Kozyrev
p-Adic numbers and complex systems

Conference #23. W. A. Zúñiga-Galindo
Non-Archimedean Reaction-Ultradiffusion Equations and Complex Hierarchic Systems

• An Ultrametric Route to Berry-Keating

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Abstract

A connection between the eigenvalue distribution of an ensemble of random matrices and the zeroes of the Riemann zeta-function has long been known. A matrix model whose phase space distribution corresponds to the non-trivial zeroes of the zeta function was recently proposed. We study matrix models corresponding to the local zeta-functions at each prime and speculate on a Berry-Keating-type Hamiltonian from its phase space description. We attempt to combine this to a Hamiltonian and a matrix model for the Riemann zeta function. This talk will be based on work in progress with A. Chattopadhyay, P. Dutta and S. Dutta.

• Random Walks And Percolation In A Hierarchical Lattice

Luis Gorostiza

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Abstract

We present some mathematical results on random walks on a class of countable ultrametric spaces called hierarchical lattices, in particular transience-recurrence properties. The motivations come from studies on populations (e.g. branching particle systems) that have a hierarchical organization. We also give some results on (long-range) percolation in such spaces, and transience-recurrence of simple random walks on percolation clusters. We make comparisons with corresponding models on Euclidean spaces.

• On p -adic string amplitudes in the limit p approaches to one

Miriam Bocado, Hugo García Compeán
(Joint work with W. A. Zúñiga-Galindo)

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Abstract

In this talk we will discuss the limit p approaches to one of tree-level p -adic open strings amplitudes through its connection with local zeta functions. There is an empirical evidence that the p -adic strings are related to the ordinary strings in the $p \rightarrow 1$ limit. In [1], we established that p -adic Koba-Nielsen string amplitudes are finite sums of Igusa's local zeta functions, and that they are convergent integrals admitting meromorphic continuations as rational functions. Denef and Loeser established that the limit $p \rightarrow 1$ of Igusa's local zeta functions give rise to new objects, that they called topological zeta functions. By using Denef-Loeser's theory of topological zeta functions, we show that limit $p \rightarrow 1$ of a tree-level p -adic open strings amplitudes give rise to a new amplitudes, which we have called string amplitudes underlying topological zeta functions. Gerasimov and Shatashvili showed that, up to second order derivatives, the effective Lagrangian in the Witten's boundary open string field theory is equal to the limit $p \rightarrow 1$ of the effective Lagrangian of the p -adic strings. In [2], we showed that the Feynman amplitudes of this last Lagrangian are precisely the string amplitudes underlying topological zeta functions, which correspond to the amplitudes of the mentioned Witten's theory.

References

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2. Bocado-Gaspar, Miriam, García-Compeán, H., W. A. Zúñiga-Galindo, *On p -adic string amplitudes in the limit p approaches to one*. Preprint 2017.

- **Ultrametric space in Teichmüller theory and nonperturbative String theory**

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Abstract

Among its many accomplishments, String Theory has a major drawback: It lacks of an exact nonperturbative formulation. In 1990 Hong and Rajeev gave insight of what a solution should be. They proposed an ill defined integration over the Universal Teichmüller space. However this space is too big (infinite non separable) to give a well defined integral. We propose as a better candidate the Teichmüller space of the Universal Hyperbolic Lamination introduced by Sullivan. We show the latter space is holomorphic and Kähler isometric to the space of continuous functions from an ultrametric space to a finite dimensional Teichmüller space and find the appropriate notions of Siegel disk and Kähler potential for this space. This is joint work with A.Verjovsky.

- **p -adic cosmology to become quantum gravity**

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Abstract

Non-local field theories originating from p -adic string theory and from string field theory form an appealing approach to resolve long-standing fundamental questions of quantum field theory. As the most notable development, non-local gravity theories including theories with non-local matter provide a challenging opportunity to explain the early universe without introducing a cosmological singularity. Moreover, the theory meets all the prerequisites to be renormalizable and unitary. As such, it is a very promising candidate for the quantum gravity. Going further one can easily embed the inflation in the non-local gravity framework as well as realize a non-singular bounce and perhaps resolve the information paradox. In my talk I will try to touch these physical aspects as much as time permits.

• p - Genetic code and Ultrametric Bioinformation

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Abstract

The genetic code is a mapping from the set of 64 codons onto the set of 20 amino acids and one stop signal. The codons are ordered triplets composed of the nucleotides cytosine (C), adenine (A), uracil (U) (or thymine (T)), guanine (G) and they are contained in the genes. The amino acids are building blocks of the proteins. The vertebrate mitochondrial (VM) code is rather simple and the other genetic codes can be considered as its slight modifications. In the VM code, an amino acid is coded by one, two or three codon doublets. When two codons code the same amino acid, they are similar (close) in the informational sense. We argue that the p -adic (ultrametric) distance is a simple, natural and adequate mathematical instrument to measure the bioinformation similarity (closeness). In particular, we show that the set of codons and the set of amino acids are p -adic ultrametric spaces and that the VM code is an ultrametric network. We also demonstrate that the (p -adic) ultrametrics is useful in investigation of informational similarity between sequences of nucleotides, codons and amino acids. This is a review talk with some new results.

• Finite automata models in Quantum Theory

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Abstract

Every automaton (a letter-to-letter transducer) whose input/output alphabets consist of p symbols produces a 1-Lipschitz map from p -adic integers to p -adic integers; and vice versa, every that map can be performed by a suitable automaton. Every that map can be regarded as a discrete causal map. By further developing of ideas of our earlier works Quantization causes waves: Smooth finitely computable functions are affine (DOI: 10.1134/S2070046615030012) and Discreteness causes waves (DOI: 10.2298/FUPCT1603143A) we derive some new results which may serve a base for future automata interpretation of Quantum Theory.

• Dynamical Systems Generated by Mappings with Delay over the p -adic Integers

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Abstract

The automaton transformation of infinite words over alphabet $\mathbb{F}_p = \{0, 1, \dots, p-1\}$, where p prime number coincide with the continuous transformation of a ring of p -adic integers \mathbb{Z}_p . The object of this study is dynamical system associated with automata maps that is important for cryptography. We prove criterion of measure-preserving for an n -unit-delay mappings associated with asynchronous automata. Moreover, we give a sufficient condition of ergodicity of such mappings.

Introduction

An automaton (letter-to-letter transducer) is tuple $\mathcal{A} = (\mathfrak{I}, \mathcal{S}, \mathcal{O}, S, O, s_0)$ where \mathfrak{I} is an input alphabet, \mathcal{S} is a set of states, \mathcal{O} is an output alphabet, $S: \mathfrak{I} \times \mathcal{S} \rightarrow \mathcal{S}$ is a state update map, $O: \mathfrak{I} \times \mathcal{S} \rightarrow \mathcal{O}$ is an output map, $s_0 \in \mathcal{S}$ is an initial state. Note that $\mathfrak{I}, \mathcal{O}$ are finite alphabets, however \mathcal{S} could be an infinite set of states.

Let's consider only accessible automata: where any state $s \in \mathcal{S}$ of automaton \mathcal{A} is reachable from initial state s_0 after a finite input word u was fed to the automaton. We assume further that $\mathfrak{I} = \mathcal{O} = \mathbb{F}_p = \{0, 1, \dots, p-1\}$, where p is prime. We identify n -letter words over \mathbb{F}_p with non-negative integers: Given an n -letter word $u = \alpha_{n-1} \dots \alpha_1 \alpha_0$, $\alpha_i \in \mathbb{F}_p$ for $i = 0, 1, 2, \dots, n-1$, we consider u as a base- p expansion of the number $\alpha_0 + \alpha_1 \cdot p + \dots + \alpha_{n-1} \cdot p^{n-1}$. In turn, the latter number can be considered as an element of the residue ring $\mathbb{Z}/p^n\mathbb{Z}$ modulo p^n . Thus, every automaton \mathcal{A} corresponds a map from $\mathbb{Z}/p^n\mathbb{Z}$ to $\mathbb{Z}/p^n\mathbb{Z}$, for every $n = 1, 2, 3, \dots$. Moreover, every automaton \mathcal{A} defines a map $f_{\mathcal{A}}$ from ring of p -adic integers \mathbb{Z}_p to itself: Given an infinite word $\alpha = \dots \alpha_{n-1} \dots \alpha_1 \alpha_0$ (that is, an infinite sequence) over \mathbb{F}_p we consider a p -adic integer x whose canonical expansion is $x = x(\alpha) = \alpha_0 + \alpha_1 \cdot p + \dots + \alpha_{n-1} \cdot p^{n-1} + \dots = \sum_{i=0}^{\infty} \delta_i(x) \cdot p^i$, where $\delta_i(x) \in \mathbb{F}_p$; so, by the definition, for every $x \in \mathbb{Z}_p$ we put $\delta_i(f_{\mathcal{A}}(x)) = O(\delta_i(x), s_i)$, $i = 0, 1, 2, \dots$ where $s_i = S(\delta_{i-1}(x), s_{i-1})$, $i = 1, 2, \dots$. We say then that map $f_{\mathcal{A}}$ is *automaton function* (or, automaton map) of the automaton \mathcal{A} .

The automaton function $f_{\mathcal{A}}: \mathbb{Z}_p \rightarrow \mathbb{Z}_p$ of the automaton \mathcal{A} is 1-Lipschitz. Conversely, for every 1-Lipschitz function $f: \mathbb{Z}_p \rightarrow \mathbb{Z}_p$ there exists an automaton $\mathcal{A} = (\mathbb{F}_p, \mathcal{S}, \mathbb{F}_p, S, O, s_0)$ such that $f = f_{\mathcal{A}}$, see [1]. The automata functions were studied in context of metric and affine equivalence of geometrical images of automata, see [4–9]. A transitive families of such mappings by means of geometrical images were described in [10].

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• A Dichotomy in p -adic Dynamics:
Measure-preservation of 1-Lipschitz functions vs
Bernoullicity of expansive functions

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Abstract

Dynamical systems on the ring \mathbb{Z}_p of p -adic integers are classified as a dichotomy between 1-Lipschitz functions and expansive functions. In this talk, we formulate a conjecture for a measure-preservation criterion of 1-Lipschitz functions on \mathbb{Z}_p , in terms of Mahler's expansion. Then we verify that it also holds for a wider class of 1-Lipschitz functions that are uniformly differentiable mod p , which is first introduced by Anashin. Also we formulate a conjecture for a Bernoullicity of expansive maps on \mathbb{Z}_p in Mahler's expansion and then verify that this conjecture holds for a wider class of expansive maps satisfying certain assumptions. The latter work is based on a Bernoullicity criterion of expansive maps on \mathbb{Z}_p in van der Put's expansion, which follows from extending the criterion of Khrennikov and Yurova.

• A review of finite approximations, Archimedean
and non-Archimedean

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Abstract

We will give review of finite approximations of quantum systems, starting with a joint paper with Varadarajan and Varadhan from 1994 (dealing with the real case), and ending with a joint paper with Bakken and Weisbart from 2017 (on stochastic methods in the non-Archimedean case).

• A heat equation on the Adéle ring

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Abstract

We introduce an ultrametric and a pseudo differential operator on the finite Adéle ring. With these tools is possible to construct a heat equation on the finite Adéle ring. Our techniques are applicable on general ultrametric spaces. Considering the place at infinity, a heat equation on the complete Adéle ring is obtained.

• Heat kernel bounds for isotropic-like Laplacians on ultrametric spaces

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Abstract

Let (X, d, m) be a proper ultrametric space equipped with a measure m . Given a symmetric measurable function $J(x, y)$ we consider the integral operator

$$L^J f(x) = \int (f(x) - f(y))J(x, y)dm(y)$$

defined on the set D of test functions, i.e. all locally constant functions f having compact support. We assume that m has full support and that the function $J(x, y)$ is uniformly in x, y comparable to a certain isotropic function $I(x, y)$. Under some reasonable assumptions on the function $I(x, y)$ the operator $(-L^J, D)$ is essentially self-adjoint, extends in $L^2(X, m)$ as a self-adjoint Markov generator and its Markov semigroup $\exp(-tL^J)$ admits a continuous transition density (heat kernel) $p^J(t, x, y)$ w.r.t. m . Moreover, the function $p^J(t, x, y)$ is uniformly comparable in t, x, y to the transition density $p^I(t, x, y)$ associated with the isotropic Markov semigroup $\exp(-tL^I)$ - an ultrametric version of the well-known Aronson's theorem for uniformly elliptic operators in euclidian spaces.

• Ultrametric models in theory of symbolic sequences

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Abstract

Motivated by possible applications of the ultrametric random processes in complex biological systems we are going to discuss a random process on a hierarchical space of states, which is described by a master equation with a modified pseudo-differential operator Q of ultrametric diffusion [1]. Contrary to the known p -adic model for description of the ligand binding kinetics in Myoglobin [2], where the diffusion is described by Vladimirov p -adic pseudodifferential operator, which, in particular, implies degeneration of the free-energy minima of the modelled system, the operator Q represents a stochastic ultrametric diffusion joined with a deterministic dynamics. The latter, to some extent, can be considered as a flux against the potential gradient. Interesting, that the above random process can be mapped on a process defined on a set of cyclic symbolic sequences. We are going to discuss the properties of such process and of the operator Q .

The symbolic sequences is a fundamental concept of modern science and technology, and widely used in such fields as information theory and bioinformatics, analysis of experimental and financial data, theory of chaos and dynamical systems. In the lecture we are going to spend some time for discussion of conceptual extensions of the above ultrametric model for applications in theory of quantum chaos and bioinformatics [3].

References

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- 3, V.Al.Osipov Wavelet Analysis on Symbolic Sequences and Two-Fold de Bruijn Sequences J. Stat. Phys. 164 (2016) 142.

- **New applications of the p -adic Nenvalinna Theory, in a field K , inside an open disk or out of a hole**

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Abstract

We establish the Nenvalinna Theory for meromorphic functions out of a hole in an ultrametric complete algebraically field K , a joint work with Tai Thi Hoai An. Motzkin Factors, known for analytic elements, play here an essential role. We also give several applications of the theory to meromorphic functions, such as sharing a finite set CM and IM and sharing two finite sets. Following results in complex analysis, we consider two analytic functions f and g in K , or inside an open disk or out of a hole, such that $f^n(x)f(x+b)$ and $g^n(x)g(x+b)$ share one point CM and prove that $f = g$ provided that $n \geq 3$ or 4.

- **Recent progress in p -adic quantum field theory**

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Abstract

A Quantum field theory over the p -adics can be seen as a probability measure on the space of Schwartz-Bruhat distributions. Gaussian measures correspond to trivial theories with no interactions. The first example of nontrivial QFT over the p -adics was constructed in 2003 by Kochubei and Sait-Ametov who treated the case of scalar theories with polynomial interactions in two dimensions and in finite volume. I will present joint work with Ajay Chandra (Warwick) and Gianluca Guadagni (Virginia) about the construction of a much more difficult example, namely, a three-dimensional scalar p -adic QFT in infinite volume. The model is a very slightly superrenormalizable in the ultraviolet and nonrenormalizable in the infrared. It is obtained by the rigorous control of a nontrivial renormalization group fixed point similar to the Wilson-Fisher fixed point. We also constructed the square of the field and showed that it has an anomalous scaling dimension which agrees with a 45-year-old prediction by Wilson. Our approach based on rigorous renormalization group methods should allow one to

prove that this model is conformally invariant in the p -adic sense. It should also provide a testing ground for the p -adic AdS/CFT correspondence which received recent attention in the work of Gubser, Marcolli and collaborators.

- **One-variable and Multi-variable Integral Calculus over the Levi-Civita Field and Applications**

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Abstract

In this talk, we present a Lebesgue-like measure theory and integration on the Levi-Civita field. After reviewing the algebraic and order structures and basic elements of calculus on the field, we introduce a measure that proves to be a natural generalization of the Lebesgue measure on the field of the real numbers and have similar properties. Then we introduce a family of simple functions from which we obtain a larger family of measurable functions and derive a simple characterization of such functions. We study the properties of measurable functions, we show how to integrate them over measurable sets of the Levi-Civita field, and we show that the resulting integral satisfies similar properties to those of the Lebesgue integral of classical Real Analysis.

Then we generalize the one-dimensional measure and integration theory to two and three dimensions, showing that the resulting measures and double and triple integrals have similar properties to those from Real Analysis. Finally, we introduce so-called delta functions which are piecewise analytic and integrable on the whole space with integral equal to 1 and which reduce to the Dirac delta function when restricted to real points; and we present simple applications of the theory.

- **p -Adic Analogue of the Porous Medium Equation**

Anatoly Kochubei

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Abstract

We consider a nonlinear evolution equation for complex-valued functions of a real positive time variable and a p -adic spatial variable. This equation is a non-Archimedean counterpart of the fractional porous medium equation. Developing, as a tool, an L^1 -theory of Vladimirov's p -adic fractional differentiation operator, we prove m -accretivity of the appropriate nonlinear operator, thus obtaining the existence and uniqueness of a mild solution. We give also an example of an explicit solution of the p -adic porous medium equation.

The talk is based on collaborations with Andrei Khrennikov, Mathematical Institute, Linnaeus University, Sweden.

- **Nonarchimedean and noncommutative aspects of the interior of the Schwarzschild black hole and signature change**

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Abstract

We consider the interior of the Schwarzschild black hole as a suitable minisuperspace cosmological model. Lagrangian of the model is transformed to two decoupled oscillators with the same frequencies and with zero energy in total. The model is presented in a p -adic and a noncommutative case. A wave function of the model is calculated, and then an adelic wave function is constructed. Signature change in p -adic and noncumulative case is also considered, followed by a discussion of the corresponding Generalized Uncertainty Principle (GUP).

- p -adic approaches to discretizing holography

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Abstract

In its most general incarnation, the AdS/CFT correspondence relates a theory of fields, including gravity, on a hyperbolic spacetime to a conformally invariant field theory on its boundary at infinity. Though the correspondence was first motivated for specific pairs of dual theories by constructions in string theory, much of the dictionary was fleshed out using general considerations about symmetries and equations of motion for fields, and the perspective has emerged that the bulk theory can be thought of as a geometrized picture of the renormalization group flow in the theory on the boundary, with the additional coordinate playing the role of a scale. Recent interest in tensor-network models has been fueled in large part by the proposal that they give a discretized analogue of an AdS/CFT-type correspondence; however, these models are not recognizably field theories, and don't exhibit the symmetry matching that is a key ingredient of the usual dictionary. I'll describe recent work that (following up on numerous clues in the literature, including a suggestion of Manin and Marcolli) begins to develop a holographic dictionary for field theories on non-Archimedean spaces, along with some new results for such field theories that have appeared along the way. Although the bulk spaces in question are automatically discrete, much of the dictionary carries over immediately from ordinary low-dimensional AdS/CFT, almost without alteration.

- Representation Theory for operators on Free Banach space of countable type

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Abstract

This talk will be centered in commutative Banach subalgebras of the algebra of bounded linear operators defined on a Free Banach spaces of countable type. The main goal of this work will be to formulate a representation theorem for these operators through integrals defined by spectral measures type. In order to get this objective, we will show

that, under special conditions, each one of these algebras is isometrically isomorphic to some space of continuous functions defined over a compact set. Then, we will identify such compact developing the Gelfand space theory in the non-archimedean setting. This fact will allow us to define a measure which is known as spectral measure. As a second goal, we will formulate a matrix representation theorem for this class of operators whose entries of these matrices will be integrals coming from scalar measures.

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• Path integrals on real, p -adic, and Adelic spaces

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Abstract

We study path integrals in ordinary, p -adic and adelic quantum mechanics for systems determined by wide class of Lagrangians. The corresponding probability amplitudes

$\mathcal{K}(x'', t''; x', t')$ for two-dimensional systems with quadratic Lagrangians are found. The obtained expressions are generalized to any finite-dimensional spaces. These exact general formulas are presented in the form which is invariant under interchange of the number fields $\mathbb{R} \longleftrightarrow \mathbb{Q}_p$ and $\mathbb{Q}_p \longleftrightarrow \mathbb{Q}_{p'}$, $p \neq p'$. This invariance shows the fundamental role of adelic path integral in mathematical physics of quantum phenomena.

2000 Mathematics Subject Classification: 58 D 30, 11 E 95.

This is joint work with Branko Dragovich.

• p -Adic numbers and complex systems

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Abstract

p -Adic and general ultrametric models can be used for modeling of complex systems. Examples of ultrametric models include applications to spin glasses, proteins, genetic code, to hierarchical clustering, and to wavelets. In particular multiclustering (a family of cluster trees constructed using a family of metrics) are related to affine Bruhat-Tits buildings and give an example of multidimensional hierarchy.

• Non-Archimedean Reaction-Ultradiffusion Equations and Complex Hierarchic Systems

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Abstract

We will discuss about non-Archimedean reaction-ultradiffusion equations and their connections with models of complex hierarchic systems. From a mathematical perspective, the equations studied here are the p -adic counterpart of the integro-differential models for phase separation introduced by Bates and Chmaj. Our equations are also generalizations of the ultradiffusion equations on trees studied in the 80 's by Ogielski, Stein,

Bachas, Huberman, among others, and also generalizations of the master equations of the Avetisov et al. models, which describe certain complex hierarchic systems. From a physical perspective, our equations are gradient flows of non-Archimedean free energy functionals and their solutions describe the macroscopic density profile of a bistable material whose space of states has an ultrametric structure. Some of our results are p -adic analogs of some well-known results in the Archimedean setting, however, the mechanism of diffusion is completely different due to the fact that it occurs in an ultrametric space. The talk will be based on reference [2].

References

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Topics

The conference will be focus on the interactions between non-Archimedean mathematics, physics, biology and computation. Among the main topics to be covered are:

- Analysis, dynamical systems and probability over non-Archimedean spaces and their applications
- Models of complex systems and biosystems
- Quantum theory, Quantum field theory and string theory in a non-Archimedean setting
- Gravity and cosmology theory in a non-Archimedean setting
- Information theory, cryptography and Bioinformatics.

The conference is organized by the Department of Mathematics of the Center for Research and Advanced Studies of the National Polytechnic Institute (Mexico) in cooperation with the Steklov Institute of Mathematics (Russia) and the International Center for Mathematical Modeling of Linnaeus University (Sweden).

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