

Recent progress in p-adic quantum field theory

Abstract

A Quantum field theory over the p-adics can be seen as a probability measure on the space of Schwartz-Bruhat distributions. Gaussian measures correspond to trivial theories with no interactions. The first example of nontrivial QFT over the p-adics was constructed in 2003 by Kochubei and Sait-Ametov who treated the case of scalar theories with polynomial interactions in two dimensions and in finite volume. I will present joint work with Ajay Chandra (Warwick) and Gianluca Guadagni (Virginia) about the construction of a much more difficult example, namely, a three-dimensional scalar p-adic QFT in infinite volume. The model is a very slightly superrenormalizable in the ultraviolet and nonrenormalizable in the infrared. It is obtained by the rigorous control of a nontrivial renormalization group fixed point similar to the Wilson-Fisher fixed point. We also constructed the square of the field and showed that it has an anomalous scaling dimension which agrees with a 45-year-old prediction by Wilson. Our approach based on rigorous renormalization group methods should allow one to prove that this model is conformally invariant in the p-adic sense. It should also provide a testing ground for the p-adic AdS/CFT correspondence which received recent attention in the work of Gubser, Marcolli and collaborators.