Exponential laws for ultrametric partially differentiable functions and applications

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I'll present exponential laws for spaces of continuously differentiable functions in several variables over a valued field $\mathbb{K}$ which have different degrees of differentiability in their arguments (the so-called C^α-functions). For example,

$$C^{(\alpha,\beta)}(U \times V, E) \cong C^\alpha(U, C^\beta(V, E))$$

if $\alpha \in (\mathbb{N}_0 \cup \{\infty\})^n$, $\beta \in (\mathbb{N}_0 \cup \{\infty\})^m$, $U \subseteq \mathbb{K}^n$ and $V \subseteq \mathbb{K}^m$ are open (or suitable more general) subsets, and E is a topological $\mathbb{K}$-vector space.

A first application concerns density questions of locally polynomial functions and polynomial functions (including the solution to an open problem by Enno Nagel). Notably, $\mathrm{Pol}(U, E)$ is dense in $C^\alpha(U, E)$ and in $C^r(U, E)$, for each locally convex space E over a complete ultrametric field $\mathbb{K}$, locally closed, locally cartesian subset $U \subseteq \mathbb{K}^n$, α as before and $r \in \mathbb{N}_0 \cup \{\infty\}$.

As a second application, one obtains a new proof for the characterization of C^r-functions on $(\mathbb{Z}_p)^n$ in terms of the decay of their Mahler expansions.

In both applications, the exponential laws enable a simple inductive proof by a reduction to the one-dimensional, vector-valued case.

References

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