

CONFERENCE PROGRAM

Hour	Monday 29 th	Tuesday 30 th	Wednesday 1 th	Thursday 2 nd	Friday 3 rd
10:00-11:00	Registration and Opening	Mini-course #1	Mini-course #1	Mini-course #1	Workshop
11:00-11:30	COFEE BREAK				TRIP TO MEXICO CITY
11:30-12:30	Conference #1	Conference #4	Conference #6	Mini-course #2	
12:45-13:45	Conference #2	Mini-course #2	Mini-course #2	Conference #8	
14:00-16:00	LUNCH				
16:00-17:00	Mini-course #1	Conference #5	Conference #7	Conference #9	
17:00-17:30	COFEE BREAK				
17:30-18:30	Conference #3	Workshop	Workshop	Conference #10	
18:30-20:00					
20:00-21:30	DINNER				

Monday 29th

Conference #1. *Spaces of analytic and differentiable functions in non-archimedean analysis*
Cristina Perez-Garcia, Cantabria University, Spain.

Conference #2. *Non Archimedean Pseudodifferential Equations of Klein-Gordon Type*
W. A. Zuñiga-Galindo, The Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico.

Minicourse #1. *Towards a New Science of Big Data Analytics, Based on the Geometry and the Topology of Complex, Hierarchic Systems*
Fionn Murtagh, Royal Holloway, University of London, UK.

Conference #3. *Linearization and periodic points in Non-archimedean dynamics*
Karl-Olof Lindah, Santiago de Chile University, Chile.

Tuesday 30th

Minicourse #1. *Towards a New Science of Big Data Analytics, Based on the Geometry and the Topology of Complex, Hierarchic Systems*
Fionn Murtagh, Royal Holloway, University of London, UK.

Conference #4. *On Some Classes of Non-Archimedean Operator Algebras*
Anatoly N. Kochubei, Institute of Mathematics, National Academy of Sciences of Ukraine.

Minicourse #2. *Introduction to Non-Archimedean Physics of Proteins*
Vladik Avetisov, Semenov Institute of Chemical Physics, Russia.

Conference #5. *Markov process on ends of tree and a nodewise orthogonal property*
Hiroshi Kaneko, Tokyo University of Science, Japan.

Workshop.

Wednesday 1th

Minicourse #1. *Towards a New Science of Big Data Analytics, Based on the Geometry and the Topology of Complex, Hierarchic Systems*
Fionn Murtagh, Royal Holloway, University of London, UK.

Conference #6. *Exponential laws for ultrametric partially differentiable functions and applications*
Helge Glöckner, Paderborn University, Germany.

Minicourse #2. *Introduction to Non-Archimedean Physics of Proteins*
Vladik Avetisov, Semenov Institute of Chemical Physics, Russia

Conference #7. *Pseudodifferential operators and parabolic type equations on adèles*
Sergii Torba, The Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico

Conference #8. *Nonlocal Equations in String Theory and Sobolev Spaces on Locally Compact Abelian Groups*
Enrique Reyes, Santiago de Chile University, Chile.

Thursday 2nd

Minicourse #1. *Towards a New Science of Big Data Analytics, Based on the Geometry and the Topology of Complex, Hierarchic Systems*
Fionn Murtagh, Royal Holloway, University of London, UK.

Minicourse #2. *Introduction to Non-Archimedean Physics of Proteins*
Vladik Avetisov, Semenov Institute of Chemical Physics, Russia.

Conference #9. *Morphisms between ultrametric Banach algebras and maximal ideals of finite codimension*
Nicolas Mainetti, Université d'Auvergne Clermont-Ferrand I, France

Conference #10. *Elementary particles and invariant measures on p -adic space-time*
Jukka Virtanen and David Weisbart, Mathematics Department, University of California Los Angeles, USA

Friday 3rd

Workshop.

ABSTRACTS

(Minicourse) Introduction to Non-Archimedean Physics of Proteins

Vladik Avetisov -Complex Systems Theoretical Lab, Semenov
Institute of Chemical Physics of the Russian Academy of Sciences,
Moscow-

It has been over 25 years since publication of the famous review "Ultrametricity for physicists" (Rammal et al. Rev. Mod. Phys. 1986) with a new philosophy in physics of spin-glasses based on hierarchy and ultrametricity. I intend to introduce the recent breakthrough of this philosophy in protein physics. Proteins often refer as complex functional systems like molecular machines, but it is still not very clear how a polymeric molecule turns into a molecular machine.

The mini-course, in its core, focuses on multi-scale description of protein dynamics by p-adic equation of ultrametric diffusion. I am going to start from two classical experiments related to protein dynamics and protein functioning, namely, the spectral diffusion in deeply frozen proteins and the CO rebinding to myoglobin. From this discussion we will get to know the puzzles coming up from the experiments and will specify where and how the problem of multi-scale description of protein dynamics appears. Next lecture will be devoted to ultrametric representation of protein energy landscape, p-adic description of protein dynamics and its physical meaning. I will focus substantially on p-adic equation of ultrametric diffusion and its solutions. Finally, on the third lecture, I will return back to the experiments and will show how physical observables are constructed under p-adic modeling. We will see that extremely complex picture of interrelated movements of thousands atoms in a protein molecule is surprisingly well described by the p-adic equation of ultrametric diffusion.

Lectures:

1. An Introduction to Proteins and Protein Dynamics

Proteins: How do they look and what they do? Two classical experiments: CO binding to myoglobin and spectral diffusion in frozen proteins. Puzzles and problems.

2. p-Adic description of multi-scale dynamics on complex energy landscapes

From tree-like presentation of high-dimensional rugged energy landscapes and basin-to-basin kinetics to ultrametric space of states, ultrametric diffusion, p-adic master equation, and the solutions.

3. Applications

Spectral diffusion in frozen proteins and first passage time distribution for ultrametric random walk. CO-rebinding to myoglobin and p-adic equations of the "reaction-diffusion" type. Concluding remarks about non-Archimedean biophysics: chromatin architecture, molecular machines, and origin of life.

Exponential laws for ultrametric partially differentiable functions and applications

Helge Glöckner -Universität Paderborn-

I'll present exponential laws for spaces of continuously differentiable functions in several variables over a valued field \mathbb{K} which have different degrees of differentiability in their arguments (the so-called C^α -functions). For example,

$$C^{(\alpha,\beta)}(U \times V, E) \cong C^\alpha(U, C^\beta(V, E))$$

if $\alpha \in (\mathbb{N}_0 \cup \{\infty\})^n$, $\beta \in (\mathbb{N}_0 \cup \{\infty\})^m$, $U \subseteq \mathbb{K}^n$ and $V \subseteq \mathbb{K}^m$ are open (or suitable more general) subsets, and E is a topological \mathbb{K} -vector space.

A first application concerns density questions of locally polynomial functions and polynomial functions (including the solution to an open problem by Enno Nagel). Notably, $\text{Pol}(U, E)$ is dense in $C^\alpha(U, E)$ and in $C^r(U, E)$, for each locally convex space E over a complete ultrametric field \mathbb{K} , locally closed, locally Cartesian subset $U \subseteq \mathbb{K}^n$, α as before and $r \in \mathbb{N}_0 \cup \{\infty\}$.

As a second application, one obtains a new proof for the characterization of C^r -functions on $(\mathbb{Z}_p)^n$ in terms of the decay of their Mahler expansions.

In both applications, the exponential laws enable a simple inductive proof by a reduction to the one-dimensional, vector-valued case.

References

- [1] Araujo, J. and W.H. Schikhof, *The Weierstrass-Stone approximation theorem for p-adic C^n -functions*, Ann. Math. Blaise Pascal **1** (1994), 61–74.
- [2] Glöckner, H., *Exponential laws for ultrametric partially differentiable functions and applications*, preprint, [arXiv:1209.1384v3](https://arxiv.org/abs/1209.1384v3).
- [3] Nagel, N., *Fractional non-Archimedean calculus in many variables*, to appear in p-Adic Numbers, Ultrametric Analysis and Applications.

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Markov process on ends of tree and a nodewise orthogonal property

Hiroshi Kaneko -Tokyo University of Science, Japan-

Stochastic processes on the field of p-adic numbers have been studied for more than two decades. The theory of Dirichlet space is applied in important

parts of the studies. In some recent observations, the hierarchical structure is handled by tree structure and the structure gives an overview for measure symmetric Markov processes on the field of p-adic numbers and ends of a tree. An orthogonal condition described in terms of Dirichlet form for eigenfunctions associated with distinct nodes is involved coherently in the existing formalisms based on Dirichlet space theory. The objective of this talk is paying a close attention to some characteristic property implied by the orthogonal condition and presenting a general formalism in compliance with the characteristic property.

On Some Classes of Non-Archimedean Operator Algebras

Anatoly N. Kochubei -Institute of Mathematics, National Academy of Sciences of Ukraine-

We study some classes of algebras of operators on non-Archimedean Banach spaces. In particular, we propose a possible way (based on the notion of a Baer ring) to develop a counterpart of the notion of a von Neumann algebra. Short of any general theory of that kind, we consider in detail a non-Archimedean version of the crossed product construction, one of the main methods of constructing von Neumann algebras in the classical case.

Linearization and Periodic Points in Non-archimedean Dynamics

Karl-Olof Lindahl - Departamento de Matemáticas y Ciencia de la Computación, Universidad de Santiago de Chile-

We discuss recent results in non-Archimedean dynamics concerning the the local geometric distribution of periodic points near indifferent fixed points and its relation to linearizability. In particular, we consider quadratic maps defined over \mathbb{C}_p . Using recent results on the size of quadratic linearization disks and localizing all periodic points of these maps we show that periodic points are not the only obstruction for linearization. In so doing, we provide the first known examples in the dynamics of polynomials over \mathbb{C}_p where the boundary of the linearization disk does not contain any periodic point.

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Morphisms between ultrametric Banach algebras and maximal ideals of finite codimension

Alain Escassut and Nicolas Mainetti

Let K be an ultrametric complete field and let E be a complete ultrametric space. Let A be the Banach K -algebra of bounded continuous functions from E to K and let B be the Banach K -algebra of bounded uniformly continuous functions from E to K . We first recall the main properties of ultrafilters, maximal ideals and multiplicative semi-norms previously studied. Next, we examine the continuous morphisms between such algebras. Finally, we show that every maximal ideal of finite codimension is of codimension 1 and under wide hypotheses we show that non-convergent ultrafilters define maximal ideals of infinite codimension.

(Minicourse) Towards a New Science of Big Data Analytics, Based on the Geometry and the Topology of Complex, Hierarchic Systems

Fionn Murtagh- Dept. of Computer Science Royal Holloway,
University of London-

This work is concerned with pattern recognition, knowledge discovery, computer learning and statistics. I address how geometry and topology can uncover and empower the semantics of data. In addition to the semantics of data that can be explored using Correspondence Analysis and related multivariate data analyses, hierarchy is a fundamental concept in this work. I address not only low dimensional projection for display purposes, but carry out search and pattern recognition, whenever useful, in very high dimensional spaces. High dimensional spaces present very different characteristics from low dimensions. It can be shown that in a particular sense very high dimensional space becomes, as dimensionality increases, hierarchical. It is also shown how in hierarchy, and hence in an ultrametric topological mapping of information space, we track change or anomaly or rupture.

Lectures:

1. An Introduction to Multivariate Data Analysis with a Focus on Hierarchical Clustering, and Correspondence Analysis - a "tale of three metrics", chi squared, Euclidean and ultrametric.
2. Data Analytics of Narrative: Pattern Recognition in Text, and Text Synthesis, Supported by the Correspondence Analysis Platform.
3. The Future of Search and Discovery in Big Data Analytics: Ultrametric Information Spaces.

4. Hierarchy and Symmetry in Data Analysis – Thinking Ultrametrically.

Spaces of analytic and differentiable functions in
non-archimedean analysis

C. Perez-Garcia- Department of Mathematics, Facultad de Ciencias,
Universidad de Cantabria, Avda. delos Castros s/n, 39071
Santander, Spain-

This talk deals with non-archimedean Köthe spaces whose Köthe duals are inductive limits. Among them we have certain spaces of analytic and differentiable functions. They play a role in theorems related to Physics. As a case in point, they are needed in the definition of a Laplace and a Fourier Transform in non-archimedean analysis. The topological properties of these Köthe spaces are crucial for the definitions of these transforms.

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Nonlocal Equations in String Theory and Sobolev
Spaces on Locally Compact Abelian Groups

Enrique Reyes -Santiago de Chile University, Chile-

In this talk I will present a rigorous analytic approach to the study of non-local equations (or, "equations in infinitely many derivatives") of interest for string theory and cosmology. In particular, I will present results on the existence of regular solutions to the generalized bosonic string equation (which involves the exponential of the Laplace operator) in Euclidean space and also in compact Riemannian manifolds. Furthermore, I will describe a recent approach to Sobolev spaces on locally compact abelian groups and I will show how this approach can be used to investigate nonlocal equations in this general context.

Pseudodifferential operators and parabolic type
equations on adèles

Sergii Torba- The Center for Research and Advanced Studies of the
National Polytechnic Institute, Mexico-

Consider the *ring of finite adèles* over \mathbb{Q} denoted \mathbb{A}_f and defined by

$$\mathbb{A}_f = \{(x_2, x_3, x_5, \dots) : x_p \in \mathbb{Q}_p, \text{ and } x_p \in \mathbb{Z}_p \text{ for all but finitely many } p\},$$

and the *ring of adèles of \mathbb{Q}* denoted \mathbb{A} and defined by

$$\mathbb{A} = \{(x_\infty, x_2, x_3, x_5, \dots) : x_p \in \mathbb{Q}_p, \text{ and } x_p \in \mathbb{Z}_p \text{ for all but finitely many } p\},$$

where \mathbb{Q}_p are the fields of p -adic numbers, \mathbb{Z}_p are the ideals of p -adic integers. Alternatively, we can define \mathbb{A}_f and \mathbb{A} as *the restricted products* of \mathbb{Q}_p with respect to \mathbb{Z}_p . The componentwise addition and multiplication give to \mathbb{A}_f and \mathbb{A} ring structures. Furthermore, \mathbb{A}_f (correspondingly, \mathbb{A}) can be made into a locally compact topological ring by taking as a base for the *restricted product topology* all the sets of the form $U \times \prod_{p \notin S} \mathbb{Z}_p$ where S is any finite set of primes (correspondingly, containing ∞), and U is any open subset in $\prod_{p \in S} \mathbb{Q}_p$.

In the talk we show that the function

$$\|x\| := \begin{cases} \max_p \frac{|x_p|_p}{p} & \text{if } x \in \prod_p \mathbb{Z}_p, \\ \max_p |x_p|_p & \text{if } x \notin \prod_p \mathbb{Z}_p \end{cases}$$

defined for arbitrary $x \in \mathbb{A}_f$ induces the non-Archimedean metric $\rho_f(x, y) := \|x - y\|$ for the ring of finite adeles. It is shown that the ring of adeles equipped with this metric becomes a complete metric space, the topology induced by the presented metric coincides with the restricted product topology and the Fourier transform of a radial function with respect to this metric is again a radial function. Moreover, balls and spheres in this metric are compact sets and their volumes are related with the second Chebyshev function.

Also we show that the metric on adeles may be introduced as

$$\rho_{\mathbb{A}}(x, y) := |x_{\infty} - y_{\infty}|_{\infty} + \|x_f - y_f\|,$$

and that this metric induces the restricted product topology.

Based on the presented metrics adelic analogues of Taibleson pseudodifferential operators and Lizorkin spaces of the second kind are introduced, their properties are discussed. For parabolic type equations involving these pseudodifferential operators the heat kernels are constructed and it is shown that a general solution of a Cauchy problem can be obtained as a convolution of the initial data with the heat kernel.

The talk is based on a joint work with W. A. Zúñiga-Galindo [1].

References

- [1] S. Torba and W. Zúñiga-Galindo, *Parabolic Type Equations and Markov Stochastic Processes on Adeles*, to appear in *Journal of Fourier Analysis and Applications*, available at arXiv:1206.5213.

Elementary particles and invariant measures on p -adic space-time

Jukka Virtanen and David Weisbart -Mathematics Department, University of California Los Angeles, USA-

We extend the method of L. Schwartz to classify elementary scalar particles in p -adic space time. In the real setting of Schwartz, elementary particles of the Poincare group correspond to invariant tempered measures supported on the

orbits of the Poincaré group. We describe an analogous notion of temperedness in the p -adic setting. In the p -adic setting, elementary particles of the generalized Poincaré group correspond to invariant measures supported on the orbits of the generalized Poincaré group. These measures are tempered in the sense we have described.

Non Archimedean Pseudodifferential Equations of Klein-Gordon Type

W. A. Zuñiga-Galindo- The Center for Research and Advanced Studies of the National Polytechnic Institute, Mexico-

We introduce a new class of non Archimedean pseudodifferential equations of Klein-Gordon type whose solutions can be easily quantized using the machinery of the second quantization. We study the Cauchy problem for these equations. We present a ‘semi-formal’ construction, on the p -adic Minkowski space, of the neutral and charged quantum scalar fields having a nonzero p -adic number as mass parameter.

References

- [1] W. A. Zuniga-Galindo , Non Archimedean Pseudodifferential Equations of Klein-Gordon Type and Quantum Scalar Fields arXiv:1302.3506